Geometry, Groups, Operator Algebras and Integrability

Abstracts of Talks

Lomonosov Moscow State University
Moscow Center for Fundamental and Applied Mathematics

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Geometry, Groups, Operator Algebras and Integrability

The conference is focused on three closely related subjects: groups and their actions in geometry and topology, operator algebras, and integrable systems. These branches of mathematics are important in themselves and have wide applications in various areas. Geometric ideas and methods play a crucial role in interconnection and development of these fields. Integrability and other properties of dynamical systems are often related with geometry and symmetries of these systems, i.e., group actions. Many operator algebras are produced from groups and bear group action, so that properties of these algebras reflect properties of the corresponding groups and vice versa. Quantum mechanics and modern physical theories use operator algebras as a basic tool. There are many other similar examples of mutual interference of these fields.

The main purpose of the conference is to bring together experts in the major fields listed in its title. We hope that this will help disseminating important ideas and broaden the areas of mutual interest.

The conference is supported by Moscow Center for Fundamental and Applied Mathematics.

The conference venue is Shuvalov Building of Moscow State University.

The web page of the conference with more details can be found at
`https://ggoi2022.mathcenter.ru`

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Abstracts

Classifying space for double points of skew-framed immersions

P. Akhmet'ev
(IZMIRAN, Russia)

A new method toward ASS (the classical Adams Spectral Sequence, [M-T]) is proposed. Our approach is based on geometrical h-principle, (the Hirsh C^0-dense principle for immersions). This method allows to detect elements in stable homotopy groups of spheres. Briefly, the C^0-dense principle allows to extends arguments by J.F.Adams on non-existence of elements of Hopf invariant one from the term E_3 to the term E_5. As an example, the following result is proved by this method.

Theorem. In the case $n = 2^\ell - 2$, $\ell \geq 7$ in the stable homotopy group of sphere Π_n there is no element with Arf-Kervaire invariant one.

In the case $n \geq 8$ this result was proved by Michael A. Hill, Michael J. Hopkins, Douglas C. Ravenel. For $n = 7$ this is a new result, conjectured by V.P. Snaith (2009).

A stable homotopy n-group of the truncated real projective space P_k is represented up to regular cobordism relation by immersions $f : M^{n-k} \longrightarrow \mathbb{R}^n$, $k \geq 1$, of a closed manifold into the Euclidean space with an additional structure Ξ of the normal bundle, which is called a skew-framing in the codimension k. The self-intersection points of f is an immersion $g : N^{n-2k} \longrightarrow \mathbb{R}^n$, equipped with an additional structure of the normal bundle Ψ, which is called a dihedral k-framing, represents an element of the stable homotopy n-group of the Thom space of the Whitney sum of k-copies of the universal $\mathbb{D}$-bundle over the Eilenberg-MacLane space $B(\mathbb{D})$.

We prove that the classifying mapping $\eta : N^{n-2k} \to B(\mathbb{D})$ admits an additional property, which is called an Abelian structure (see Definition 6 [A]). To formulate this property we construct an universal space for double points of skew-framed immersions, using a moments-angles complex.

At the second step, we consider immersions $h : L^{n-8k} \longrightarrow \mathbb{R}^n$, which are called 2-iterated self-intersections of skew-framed immersion with Abelian structure. Standardly, such an immersion is equipped with the structuring mapping $\zeta : L^{n-8k} \to B(\mathbb{Z}/2^{[4]})$, where $\mathbb{Z}/2^{[4]}$ is the generalized dihedral group of the order 2^{15}. We proved that an additional structure for the mapping ζ, called the bi-cyclic structure, is well-defined, see Definition 14,15 [A]. A bi-cyclic structure is a natural lift of the structuring mapping to a mapping $L^{n-8k} \longrightarrow B((\mathbb{Z}/4 \times \mathbb{Z}/4) \rtimes \mathbb{Z}^3)$, where the target space is the Eilenberg-MacLane space $B(\mathbb{Z}/4) \times B(\mathbb{Z}/4)$ over an order 3 Laurent extension.

Arguments using desuspension theorem as in [R-L], homology groups with an order 3 tower of local coefficients, and a modification of the Herbert Theorem for self-intersection of immersions prove the Theorem.

Our approach develops approaches by L.S.Pontrjagin and by V.A.Rokhlin. An idea to investigate additional properties of self-intersection of immersions was communicated to me by A.V.Chernavskii (1996). S.A.Melikhov (2005) has noted that a quaternion analogue of the Chernavskii construction is required. An idea to make a C^0-dense control over the projective plane was realized after a talk by E.V.Scepin (2004) before his business trip. As a conclusion, results were collected-out at the A.S.Mishchenko Seminar in (2008-2022). The result is particularly based on common results with Th.Yu.Popelenskii.

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Poisson brackets and flat connections

A. Alekseev

(Université de Genève, Switzerland)

A flat connection with regular singularities on a complex plane is characterized by its residues in singular points. A theorem of Hitchin establishes a surprizing link between the linear Kirillov-Kostant-Souriau (KKS) Poisson bracket on the residues and the Goldman bracket on traces of holonomies corresponding to closed loops. In this talk, we will show how to generalize this result to regularized holonomies corresponding to paths starting and ending at singular points. Our main technical tool is a version of the Drinfeld's pentagon equation suitable for paths with self-intersections.

The talk is based on a joint work in progress with Florian Naef and Muze Ren.

Banach algebras of polynomial growth and universal algebras of non-commutative C^∞-functions

O. Aristov

(Moscow, Russia)

We are interested in topological algebras whose elements can be treated as C^∞-functions on some NC spaces. It is natural to assume that functions of class C^∞ operate on such algebras Traditionally, certain subalgebras of C^*-algebras are considered but I propose an alternative approach based on the class PGL, projective limits of Banach algebras of polynomial growth (over the field of real numbers). Polynomial growth means that for every element b the norm of $s \mapsto exp(isb)$ grows more slowly than a polynomial in $|s|$ (here s is real). This condition is equivalent to the existence of C^∞-functional calculus. Operators of polynomial growth are known since the early 60s but Banach algebras consisting entirely of such operators have not been studied before. Note that we do not assume that the algebra contains in a C^*-algebra but all known examples can be approximated by operator algebras (non-selfadjoint in the NC case). Algebras in PGL are commutative modulo Jacobson radical and so admit only one-dimensional irreducible representations. Nevertheless, this class turns out to be unexpectedly wide and contains many interesting examples, such as algebras associated with Lie algebras, free algebras and C^∞-versions of some quantum groups. The simplest are $C^\infty(\mathbb{R})$ and the algebra of triangular matrices. It is the non-triviality of the radical that is the main feature of most interesting examples since it is the radical that is responsible for the non-commutativity. For a real topological algebra we consider the universal algebra closest to it in PGL, the envelope in the class of Banach algebras of polynomial growth. In the case when initial algebra is finitely generated the envelope is an example of a finitely C^∞-generated algebra. Such algebras are of the greatest interest because they can be treated as algebras of 'smooth functions' on a NC C^∞-differentiable spaces. In particular, being close to commutative, a finitely C^∞-generated algebra admits (at least in some cases) a structure sheaf of NC functions.

Derivations on bimodules

A. Arutyunov

(V. A. Trapeznikov Institute of Control Sciences of Russian Academy of Sciences, Russia)

We will talk about a combinatorial method for studying derivations on bimodules that arise as a completion of a group ring with respect to various norms. It will be shown that for a wide class of norms all derivations are quasi-inner. The question of the coincidence of quasi-inner and inner derivations depending on the internal structure of the group will also be studied.

The work is a development of the results obtained jointly with Prof. A.S. Mishchenko and Prof. A.I. Shtern.

Powers of sets in periodic groups

V. Atabekyan
(Yerevan State University, Armenia)

It is proved that for any finite symmetrized subset S of the free Burnside group $B(m,n)$ the inequality $|S^t| \geq 4 \cdot 2.9^{t/(400s)^3}$ holds, where s is the smallest odd divisor of n that satisfies the inequality $s \geq 1003$.

Courant Algebroids and Representations

P. Batakidis
(Aristotle University Thessaloniki, Greece)

Courant algebroids is a generalization of quadratic Lie algebras, among many others. In this talk we will discuss the main differential geometric constructions leading to the notion of representations of Courant algebroids through linear and non-linear connections of them. In parallel, we explain the relation to Manin pairs, that is Courant algebroids coupled with Dirac structures. Typical examples include complex manifolds, foliations, Poisson and almost symplectic structures and etc.

Geometry of cotangent bundle of Heisenberg group

N. Bokan
(University of Belgrade - Faculty of Mathematics, Serbia)

The cotangent bundles play a significant role in standard description of physical systems, both for particles and for fields. They appear as the configuration space of some mechanical systems and are frequently endowed with rich algebraic structures. We are interested in the cotangent bundle $T^*\mathfrak{h}_3$ of the Heisenberg group H_3, mainly because this group is a constant topic of research due to its properties and various areas of applications.

First, we obtain all orbits of moduli space $\mathfrak{M}(T^*\mathfrak{h}_3)$ of left invariant metrics on $T^*\mathfrak{h}_3$ under the action of the group $Aut(T^*\mathfrak{h}_3)$. It is shown that some of these orbits are closely related to very old results concerning the classification of conics in hyperbolic plane, studied by B. Rosenfeld and B. Wiebe, W.E. Story, K.A. Umlauf, F.Klein and W. Roseman. To describe all orbits we use also various algebraic and geometric approaches in compliance with the nature of $T^*\mathfrak{h}_3'$. It is shown that geometrical properties of this moduli space are very rich. We describe all geodesically equivalent metrics by using results of G.I. Kruchkovich and A.S. Solodovnikov. The classification of pseudo-Kähler and *pp*-wave metrics, as well as algebraic Ricci solitons, are described. We compute curvatures for all representative metrics of orbits in $\mathfrak{M}(T^*\mathfrak{h}_3)$ to describe their holonomy algebras using the Ambrose-Singer theorem. Subalgebras of $T^*\mathfrak{h}_3$ being totally geodesic are also investigated.

This is joint work with Tijana Šukilović and Srdjan Vukmirović.

Holomorphic solutions of integrable evolution equations

A. Domrin
(Lomonosov Moscow State University, Russia)

A local holomorphic version of the inverse scattering transform enables us to construct all local holomorphic solutions of soliton equations of parabolic type and study their properties. In particular, we prove that every such solution extends to a global meromorphic function of the spatial variable. We also describe all integrable evolution equations with weighted homogeneous right-hand side all of whose local holomorphic solutions possess this forced analytic extension property.

On a generalization of the topological Brauer group

A. Ershov
(MIPT, Russia)

In the talk I will give a geometric description of "higher" twistings of topological K-theory that have finite order. For this purpose we consider locally trivial bundles with fiber $M_{kl^\infty} = \varinjlim_n M_{kl^n}(\mathbb{C})$ and structure group $\mathrm{Aut}(M_{kl^\infty})$, where $gcd(k,l) = 1$. There is a natural transformation induced by the assignment $A_k \mapsto A_k \otimes M_{l^\infty}$, where $A_k \to X$ is a locally trivial $M_k(\mathbb{C})$-bundle. We show that this transformation trivializes exactly those $M_k(\mathbb{C})$-bundles that admit a unital embedding into a trivial bundle $X \times M_{kl^n}(\mathbb{C})$ for some n. This allows us to prove that there is a $2k$-equivalence between the direct limit of homogeneous spaces $\mathrm{PU}(kl^n)/(E_k \otimes \mathrm{PU}(l^n))$ and the topological group $\mathrm{Aut}(M_{kl^\infty})$. Then we define the generalized Brauer group of X as the group of equivalence classes of $M_{k^n l^\infty}(\mathbb{C})$-bundles over X modulo those that have the form $\mathrm{End}(\xi_{k^n}) \otimes M_{l^\infty}(\mathbb{C})$ for some $\mathbb{C}^{k^n}$-vector bundle $\xi_{k^n} \to X$. We also show that the classical Brauer group is a direct summand of the generalized one.

Dispersionless integrable equations and modular forms

E. Ferapontov
(Loughborough University, UK)

It has been observed that coefficients of (generic) dispersionless integrable PDEs within various particularly interesting classes are expressed via modular forms. In this talk, I will discuss two particular instances of this phenomenon:

1. Integrable equations of the dispersionless Hirota type where generic case coincides with the equation of the genus three hyperelliptic divisor.

2. First order integrable Lagrangians where generic case is expressed via Picard modular forms.

This talk is based on joint work with F. Clery, A. Odesskii and D. Zagier.

Satake correspondence and cluster duality

V. Fock

(Université de Strasbourg, France)

Satake correspondence is an isomorphism between the representation ring of a finite dimensional Lie group and the Hecke algebra corresponding to the dual affine Lie algebra. On the other hand the space of integer measured laminations generate an algebra isomorphic to the space SL(2) local systems. We going to suggest a construction interpreting higher laminations as local systems with values in an affine Weyl group and relating both isomorphisms as a particular cases of cluster duality.

The cohomology rings of homogeneous spaces

M. Franz

(University of Western Ontario, Canada)

Let G be a compact connected Lie group and K a closed connected subgroup. Assume that the order of any torsion element in the integral cohomology of G and K is invertible in a given principal ideal domain k. It is known that in this case the cohomology of the homogeneous space G/K with coefficients in k and the torsion product of $H^*(BK)$ and k over $H^*(BG)$ are isomorphic as k-modules. We show that this isomorphism is multiplicative and natural in the pair (G, K) provided that 2 is invertible in k. The proof uses homotopy Gerstenhaber algebras in an essential way.

An answer to a question of J.W. Cannon and S.G. Wayment

O. Frolkina

(Lomonosov Moscow State University, Russia)

Solving R.J. Daverman's problem, V.S. Krushkal described sticky Cantor sets in $\mathbb{R}^N$ for $N > 3$; these sets cannot be isotoped off of itself by small ambient isotopies. Using Krushkal sets, we answer a question of J.W. Cannon and S.G. Wayment (1970). Namely, for $N > 3$ we construct compacta X in $\mathbb{R}^N$ with the following two properties: some sequence of compacta X_k in $\mathbb{R}^N \setminus X$ converges homeomorphically to X, but there is no uncountable family of pairwise disjoint subsets of $\mathbb{R}^N$ each of which is embedded equivalently to X.

Counterexamples in Hilbert C*-modules theory

D. Fufaev
(Moscow Center of Fundamental and Applied Mathematics, Lomonosov MSU, Russia)

We will discuss some statements and examples showing the specifics of Hilbert C*-modules.

Free actions of discrete groups on even dimension homotopy spheres

D. L. Gonçalves
(Institute of Mathematics and Statistics - University of São Paulo, Brasil)

Let Σ^{2n} be a finite dimensional CW complex which has the homotopy type of the sphere S^{2n} (in short, a homotopy $2n$-sphere). In this talk we begin by describing the discrete groups G which act on such spheres. It turns out that under the hypothesis that the group has finite virtual cohomology dimension ($vcd(G) < \infty$), these groups are of the form $G = G_0 \rtimes Z_2$ where G_0 is torsion free and has finite cohomology dimension, i.e. $cd(G_0) < \infty$. Then we move to a question, which in the case of finite G is well understood. Namely a given action of the group $G \times \Sigma^{2n} \to \Sigma^{2n}$ provides a representation $\phi : G \to Aut(H^{2n}(\Sigma^{2n}, Z))$ or $\phi : G \to Aut(Z) = Z_2$. We provide a necessary condition that this homomorphism ϕ is realizable, in terms of the Farell cohomology of G. Then we present two families of examples, which are families of pairs (G, φ) which can be realized. The second family is new and G_0 is the mapping class group of a closed surface.

This is joint work with Sérgio Tadao Martins.

Below we give some relevant (but not all) references closely related to this talk:

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Double-periodic rogue waves type solutions of the focusing Davey-Stewardson-2 equation: the finite-gap approach

P. Grinevich

(Steklov Mathematical Institute, Moscow, Russia)

Recently in a series of papers we developed finite-gas approach to the periodic anomalous waves Cauchy problem for the focusing Nonlinear Schrodinger equation. Due to the presence of a small parameter in this problem the leading order approximation of the finite-gap solutions can be written in terms of elementary functions (and remains highly non-linear).

In the present work we derive analogous elementary formulas for the focusing Davey-Stewardson 2 equation, which is an integrable 2+1 systems admitting rogue-waves type solutions. This is a joint work with P.M. Santini.

Chevalley type theorem for Jacobi modular forms and non-linear differential equations

V. Gritsenko

(Université de Lille, HSE University, Moscow)

Jacobi modular forms appear in many problems in number theory, geometry, topology, the theory of Kac-Moody algebras and mathematical physics. Jacobi modular forms of weight 0 and -2 are elliptic genera of Calabi-Yau varieties, generating functions of multiplicities of positive roots of Lorentzian Kac-Moody algebras and Gromov-Witten invariants. The generators of polynomial rings of Weyl invariant weak Jacobi forms are used for construction of Frobenius manifolds. For the root lattice D_n with n smaller than 9 the corresponding Jacobi forms in n abelian variables generate the reflective automorphic Borcherds products related to BCOV-analytic torsion.

With Dimitri Adler, we generalize the idea of the D8-tower of the reflective Jacobi forms, and give a constructive proof of the Chevalley type theorem for invariant Jacobi forms for the root systems C_n and D_n for arbitrary n. Our method gives a system of modular differential equations, which is satisfied by three main generators of index 1 and a non-linear differential equation for each generator. A surprise is that there are unusual anomalies for some small n, specially for the Jacobi form of weight -2 of Gromov-Witten type.

Any suspension and any homology sphere are $2H$-spaces

D. Gugnin

(Lomonosov Moscow State University, Russia)

Recall that a path-connected Hausdorff space X with a basepoint $e \in X$ is an H-space if there exists a continuous multiplication $\mu\colon X \times X \to X$ such that $\mu(x, e) = \mu(e, x) = x$ for all $x \in X$. Here we do not require homotopy associativity (and an inverse map), so we consider the notion of an H-space in the wide sense. Hopf invariant one theorem states that a sphere S^m is an H-space iff $m = 1, 3, 7$.

There exists a generalization of this notion, first introduced (implicitly) by Buchstaber in 1990: a path-connected Hausdorff space X with a basepoint $e \in X$ is an nH-space, $n \geq 2$, if there exists a continuous n-valued multiplication $\mu\colon X \times X to \mathrm{Sym}^n X$ such that $\mu(x, e) = \mu(e, x) = [x, x, \ldots, x]$ for all $x \in X$. The space $\mathrm{Sym}^n X$ is just an n-th symmetric product X^n/S_n of a space X.

It is evident that any H-space is trivially an nH-space for all $n \geq 2$. The first nontrivial example of a $2H$-space is a 2-sphere S^2 (Buchstaber, 1990). The author in 2019 proved that all odd-dimensional spheres $S^{2k+1}, k \geq 0$, are $2H$-spaces (the argument used the Whitehead product on homotopy groups of spaces).

The main results to be presented at the talk are the following three theorems.

Theorem 1. *For any connected finite or countable polyhedron Y its reduced suspension $X = \Sigma Y$ admits a structure of an nH-space for all $n \geq 2$.*

Theorem 2. *All smoothable homology spheres $\Sigma^m, m \geq 3$, admit a structure of an nH-space for all $n \geq 2$.*

Theorem 3. *Any finite connected CW-complex X with $H_1(X; \mathbb{Z}) = 0$ admits a structure of an nH-space for all $n \geq \dim X$.*

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Mirror symmetry on invariants of the Berglund-Hübsch-Henningson-dual non-commutative orbifolds and its distribution among levels

S. Gusein-Zade

(Lomonosov Moscow State University, Russia)

The Berglund-Hübsch-Henningson duality was the first systematic attempt to construct mirror symmetric Landau-Ginzburg models. The initial data for an (orbifold) Landau-Ginzburg model is a pair (f, G) consisting of a quasihomogeneous polynomial f in several variables and a finite group of linear transformations preserving f. In the Berglund-Hübsch-Henningson construction f was a so-called invertible polynomial and G was a group of its diagonal symmetries. (A quasihomogeneous polynomial f is called invertible if the number of monomials in it is equal to the number n of variables, i.e. $f(\overline{x}) = \sum_{i=1}^{n} a_i \prod_{j=1}^{n} x_j^{E_{ij}}$, $a_i \neq 0$ and $\det(E_{ij}) \neq 0$. Without loss of generality one may assume that $a_i = 1$.) From a pair (f, G) of the described type one constructs a dual pair $(\widetilde{f}, \widetilde{G})$. (One has $\widetilde{f}(\overline{x}) = \sum_{i=1}^{n} \prod_{j=1}^{n} x_j^{E_{ji}}$.) Dual pairs (f, G) and $(\widetilde{f}, \widetilde{G})$ possess a number of "mirror symmetry" properties (for example, a symmetry of a number of orbifold invariants, the simplest of whom is the orbifold Euler characteristic).

This duality was extended to pairs $(f, \hat{G})$, where f is an invertible polynomial and $\hat{G}$ is the semidirect product of a group G of diagonal symmetries of f and a group S of permutations of the coordinates preserving f and G. The construction is based on ideas of A.Takahashi and therefore is called the Berglund-Hübsch-Henningson-Takahashi duality. It appears that dual pairs can pretend to be mirror symmetric only under a special restriction on the grpoup S called the parity condition. For some dual pairs satisfying the parity condition, there were proved symmetries of such invariants as the orbifold Euler characteristic, orbifold monodromy zeta-function, orbifold E-function. One has the conjecture that the invariants under consideration are split into summands (or factors) corresponding to the conjugacy classes of elements of S which possess the same symmetries (if the parity condition holds). This conjecture was verified for some cases.

The talk is based on joint results with W. Ebeling.

Asymptotic enumerations of threshold functions and singular ± 1-matrices

A. Irmatov

(Lomonosov Moscow State University, Russia)

Two results concerning asymptotics of the number of threshold functions and asymptotics of the probability that a random $n \times n$ Bernoulli matrix is singular established by author will be discussed.

Noncommutative coverings and prospects of their applications to theoretical physics

P. Ivankov

(Lomonosov Moscow State University, Russia)

According to the Gelfand-Naimark theorem any C^*-algebra is a noncommutative generalization of a topological space. Moreover, there are noncommutative analogs of topological coverings which can be included into a following list:

- Coverings of C^*-algebras of foliations.
- Coverings of noncommutative tori.
- Covering of the quantum version of SO(3).

This work is devoted to a single general theory which includes all theories of this list, i.e. we develop a system of axioms which can be applied for every element of the list. We consider prospects of their applications to the solving of following problems:

- Black hole singularity.
- Renormalization divergences.

Recent progress in ellipsoid characterization problems

S. Ivanov

(St. Petersburg Department of Steklov Mathematical Institute, Russia)

I will speak about classical results, problems, and recent developments on characterizations of Euclidean spaces among finite-dimensional Banach spaces. I will concentrate on an old problem asking whether an n-dimensional Banach space is necessarily Euclidean if, for some fixed $1 < k < n$, all k-dimensional linear subspaces are isometric. Equivalently, the question asks whether an n-dimensional centered convex body all whose k-dimensional central cross-sections are linearly equivalent, is necessarily an ellipsoid. The answer is known to be affirmative in some dimensions and unknown in others. Among other things I will announce a solution (joint with D.Mamaev and A.Nordskova) for $n = 4$ and $k = 3$.

The talk is based on a joint work in progress with Florian Naef and Muze Ren.

Degenerate singularities of mechanical systems

V. Kibkalo

(Lomonosov MSU, Moscow Center for Fundamental and Applied Mathematics, Russia)

Classification of singularities of integrable systems is essencial for this branch of mathematics: neighbourhood of a regular fiber is completely described by the Liouville-Arnold theorem. Topological classification of semi-local corank-1 nondegenerate singularities (the Morse-Bott singluarities) was obtained by A.Fomenko and used to define more sophisticated global invariants of Liouville foliations (e.g., Fomenko-Zieschang invariants). Such invariants were calculated for a wide class of integrable systems from mechanics, mathematical physics and geometry. Introduced by L.Lerman as a transition from a family of nondegenerate saddle orbits to the one of nondegenerare elliptic one, parabolic singularities (orbits) seems to be one of the most simple example of degenerate singularities of integrable systems. Differential geometric criterion an orbit to be parabolic was suggested in a recent paper by A.Bolsinov, E.Kudryavtseva and L.Guglielmi. Also the structural stability of parabolic points (orbits) and cuspidal tori under small integrable perturbations of a system was proved. As it turnes out, such singularities indeed appear in integrable systems of mechanics and their analogs on Lie coalgebras. For several systems (Zhukovskii axi-symmetric case, Kovalevskaya top and its analogs on Lie coalgebras $so(3,1)$ and $so(4)$), all such singularities were calculated. Some of obtained results are joint with E.Kudryavtseva.

Semiclassical trace formulas for the Bochner Laplacian on a Hermitian line bundle

Yu. Kordyukov

(Institute of Mathematics, Ufa Federal Research Centre, Russia)

The talk is devoted to the study of the asymptotic behavior of the spectrum of the Bochner Laplacian on high tensor powers of a Hermitian line bundle on a compact Riemannian manifold. We discuss semiclassical trace formulas, which give an asymptotic description of the smoothed eigenvalue distribution function of the operator for certain intervals of the real line in terms of the associated classical dynamics. The talk is partially based on a joint work with I. A. Taimanov.

On the Gröbner–Shirshov bases for vertex operator algebras

R. Kozlov
(Lomonosov MSU, Moscow Center for Fundamental and Applied Mathematics, Russia)

The notion of a vertex (operator) algebra was introduced by R. Borcherds as an algebraic language of describing the operator product expansion (OPE) of chiral fields in 2d conformal field theory. Later V. Kac introduced conformal algebras as algebraic formalization of the singular part of the OPE. There are many examples of vertex algebras known in literature, mainly appearing in physics and representation theory. A typical method to define a vertex algebra is to take a vector space V and set up a collection of vertex operators – infinite power series in both positive and negative exponents with coefficient in $\mathrm{End}(V)$ – and then check the conditions of the Goddard Uniquness Theorem, which is quite a task even in the simplest cases. In the same time, in many "good" varieties of algebras, e.g. associative, Lie algebras, we are able to work with quotients of the free object modulo a congruence generated by some defining relations. Finding a description of a system defined in this fashion may turn to be a tough combinatorial problem though, this approach could be of use in many cases. For associative and commutative algebras, the technique of Gröbner bases is a routine way to determine normal forms of elements in a quotient of polynomial algebras. Nowadays, the bunch of techniques based on Newmann's Diamond Lemma for finding normal forms in various algebraic systems is known as Gröbner-Shirshov bases (GSB) method. This method also works for vertex algebras without an additional complexification. In our talk, we observe the "pure algebraic" definition of a vertex algebra, presented by B. Bakalov and V. Kac, and describe the GSB approach to the problem of finding the structure of a vertex algebra defined by generators and relations. As an application, we calculate GSBs as for naturally appeared classical vertex (super)algebras so as for artificial ones, in sense of the previous sentence.

Higher local systems, loop spaces and derived categories of second kind

A. Lazarev
(Lancaster University, UK)

It is well-known that a local system on a manifold M can be specified as a representation of the fundamental group of M, or as a flat connection on a vector bundle on M. The data of a flat connection can be encoded in the form of a twisted de Rham complex, viewed as a module over the ordinary de Rham algebra of M of a special kind. In this talk I will explain what happens if one considers graded flat connections and the homotopy category of the corresponding twisted modules. Namely, this category can be interpreted as the category of higher local systems on M, or cohomologically locally constant sheaves on M. There exists also an analogue of this construction where M is not necessarily a manifold, and the role of the algebra A is played by the singular cochain algebra of M.

This is joint work with J. Chuang and J. Holstein.

Arithmetic squares and demystification of J.-P. Serre's philosophy

A. Mikhovich
(Moscow Center for Fundamental and Applied Mathematics, Russia)

Extending Sullivan's results, we show that the Bousfield-Kan integral completion of cellular spaces with a finite number of non-degenerate cells in each dimension is calculated as the homotopy inverse limit of the arithmetic square of p-adic and rational completions. In particular, the homotopy groups of the Bousfield-Kan integral completion of such spaces are computable from the homotopy groups of p-adic and rational completions using the Mayer-Vietoris sequence for homotopy groups.

This observation allows us to explain the philosophy of J.-P. Serre that, in problems of a homological nature, the homotopy properties of presentations of discrete groups are determined by the properties of the corresponding pro-p-presentations.

As an application to the Whitehead's asphericity problem, we prove that the Bousfield-Kan integral completion of a sub-presentation of a contractible discrete presentation is aspherical.

Einstein derivations and narrow Lie algebras

D. Millionshchikov
(Lomonosov Moscow State University, Russia)

Consider a nilpotent Lie group G with a left invariant Riemannian metric g. The metric g defines the Euclidean inner product on the Lie algebra $\mathfrak{g}$ of G. The left-invariant Ricci tensor Ric of the metric g defines a self-adjoint Ricci operator $R : \mathfrak{g} \to \mathfrak{g}$. Fixing some orthonormal basis $e_1, \dots, e_n$ in the metric *nilpotent* Lie algebra $\mathfrak{g}$, one can write R as

$$R = \frac{1}{4}\sum_{i=1}^{n} ad_{e_i} ad_{e_i}^* - \frac{1}{2}\sum_{i=1}^{n} ad_{e_i}^* ad_{e_i},$$

where $ad_{e_i}^*$ denotes the adjoint operator to $ad_{e_i}, ad_{e_i}(x) = [e_i, x]$.

The operator R, like the identity operator Id, is not a derivation of the Lie algebra $\mathfrak{g}$, but it may happen that for some constant $c \in \mathbb{R}$ the operator $D = R - cId$ will be a derivation of the Lie algebra $\mathfrak{g}$ and in this case the derivation D is called the *Einstein derivation* of the Lie algebra $\mathfrak{g}$ [1].

J. Heber proved in 1998 that all eigenvalues λ_i of the Einstein derivation D, up to scaling, are positive integers and hence the Lie algebra $\mathfrak{g}$ is positively graded [2].

In the talk, we will be especially interested in the case when all eigenvalues λ_i have multiplicity one, which means that our Lie algebra $\mathfrak{g}$ is narrow in the sense of Zelmanov and Shalev [4].

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Looking at Poisson manifolds through new symplectic glasses

E. Miranda

(Universitat Politècnica de Catalunya — BarcelonaTech, Spain)

b-Structures and other generalizations (such as E-symplectic structures) are ubiquitous and sometimes hidden, unexpectedly, in a number of problems including the space of pseudo-Riemannian geodesics and regularization transformations of the three-body problem. E-symplectic manifolds include symplectic manifolds with boundary, manifolds with corners, compactified cotangent bundles and regular symplectic foliations. But how general can such structures be? In this talk, I will explain how to associate an E-symplectic structure to a Poisson structure with transverse structure of semisimple type (joint work with Ryszard Nest). This should enable us to address a number of open questions in Poisson Geometry and Hamiltonian Dynamics from a brand-new perspective.

Comparison of Hochschild homology and cohomology of group algebras

A. Mishchenko

(Lomonosov Moscow State University, Russia)

In addition to numerous abstract examples generalizing cohomology with compact supports, there are, perhaps, the only concrete example in mathematics that represents nontrivial supports families. Such an example gives us the Hochschild cohomology of the group algebra $R[G]$, which are isomorphic to the classical cohomology of the classifying space BGr of the groupoid Gr associated with the attached action of the group G. These Hochschild cohomology of the group algebra $R[G]$, are isomorphic to the classical cohomology of the classifying space BGr of the groupoid Gr, whose supports belong to specific families of supports Φ_n in n-sceletons $(BGr)[n]$ of the classifying space BGr of the groupoid Gr generated by the adjoint action of the group G: $HH^n(R[G]) \cong H^n\Phi_n(BGr; R)$.

Affine center at the critical level and quantum Mishchenko-Fomenko subalgebras

A. Molev

(University of Sydney, Australia)

We consider the affine vertex algebra at the critical level associated with a simple Lie algebra g. We obtain new uniform expressions for generators of the center of this vertex algebra for types A,B,C,D. The results are then applied to calculate the Harish-Chandra images of the Casimir elements constructed via the canonical symmetrization map from the basic invariants in the symmetric algebra S(g). As another application, we produce explicit generators of the quantum Mishchenko-Fomenko subalgebras of the universal enveloping algebra U(g).

Uniformization, symplectic reduction, and asymptotic solutions

V. Nazaikinskii

(Ishlinsky Institute for Problems in Mechanics, Moscow, Russia)

We illustrate the general ideas of uniformization and symplectic reduction in asymptotic theory by the example of nonlinear shallow water equations describing gravity waves in a bounded basin with a smooth depth function $D(x)$ whose gradient nowhere vanishes on the coastline $D(x) = 0$. For solutions of small amplitude, it is natural to treat these equations as a perturbation of the linearized equations, but there are two obstructions to a straightforward application of regular perturbation theory: (i) the solution (η, v) of the nonlinear equations (here η is the free surface elevation and v is the velocity) is defined in the domain $\Omega_\eta = \{D(x) + \eta(x) > 0\}$, which depends on the solution itself, while the linearized equations are naturally considered in the fixed domain $\Omega = \{D(x) > 0\}$; (ii) the linearized equations degenerate on $\partial\Omega$. To tackle (i), one uses a solution-dependent change of variables (resembling the well-known Carrier–Greenspan transformation) that takes Ω_η to Ω. Now we can proceed to the analysis of the linearized equations.

Uniformization. We represent the closure of Ω as the quotient of a closed 3-manifold X by a circle action, the boundary being the image of the fixed-point set of the action, and lift the linearized equations to X. The lifted equations prove to be hypoelliptic, which justifies the use of regular perturbation theory. The leading term of the solution of the nonlinear equations is obtained from the solution of the linearized equations by the above-mentioned change of variables.

Symplectic reduction. Asymptotic solutions of the linearized equations, in turn, can be obtained with the use of Maslov's canonical operator. However, the Lagrangian manifold on which the canonical operator is defined lies in a symplectic manifold Φ obtained from T^*X by symplectic reduction rather than in $T^*\Omega$. Hence the construction of the canonical operator turns out to be nonstandard; in particular, it is based on the Bessel transform rather than the Fourier transform in a neighborhood of the boundary.

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Many faces of Calogero-Moser system as seen by sigma models and gauge theories

N. Nekrasov

(Stony Brook University, USA)

I will review some less known facts about the elliptic and hyperbolic CM many-body system and its connections to supersymmetric sigma models in two dimensions and gauge theory in $\approx$ four.

On the index of G-invariant pseudodifferential operators

V. Nistor

(Institut Élie Cartan de Lorraine, Nancy, France)

Let G be a compact Lie group acting on a smooth manifold M (without boundary). Let $E \to M$ be an equivariant bundle and P be a G-invariant pseudodifferential operator acting on the sections of E. Let α be an irreducible representation of G and $\pi_\alpha(P)$ be the restriction of P to the isotypical component corresponding to α. We study when this operator is Fredholm and the resulting algebra of symbols. We give a simple, necessary and sufficient criterion for $\pi_\alpha(P)$ to be Fredholm. We also provide a spectral sequence converging to the periodic cyclic homology of the corresponding algebra of symbols. This work was done in collaboration with A. Baldare, M. Benameur, R. Côme, and M. Lesch.

On the sheaf of smooth forms on Lie algebroids over triangulated spaces

J. Oliveira

(University of Minho, Portugal)

A sheaf of differential forms on a compatible family of Lie algebroids defined over regular open subsets of a simplicial complex is constructed and is proved that sheaf is fine.

Holomorphic foliations on complex moment-angle manifolds

T. Panov

(Lomonosov Moscow State University, Russia)

Moment-angle manifolds provide a wide class of examples of non-Kähler compact complex manifolds with a holomorphic torus action. A complex moment-angle manifold Z is constructed via a certain combinatorial data, called a complete simplicial fan. In the case of rational fans, the manifold Z is the total space of a holomorphic bundle over a toric variety with fibres compact complex tori. In this case, the invariants of the complex structure of Z, such Dolbeault cohomology and the Hodge numbers, can be analysed using the Borel spectral sequence of the holomorphic bundle.

In general, a complex moment-angle manifold Z is equipped with a canonical holomorphic foliation F which is equivariant with respect to the algebraic torus action. Examples of moment-angle manifolds include the Hopf manifolds, Calabi–Eckmann manifolds, and their deformations. The holomorphic foliated manifold (Z, F) has been also studied as a model for irrational ("non-commutative") toric varieties in the works by several authors (arXiv:1308.2774, arXiv:2002.03876).

We construct transversely Kähler metrics on moment-angle manifolds Z, under some restriction on the combinatorial data. We prove that all Kähler submanifolds in such a moment-angle manifold lie in a compact complex torus contained in a fibre of the foliation F. For a generic moment-angle manifold Z in its combinatorial class, we prove that all its subvarieties are moment-angle manifolds of smaller dimension. This implies, in particular, that Z does not have non-constant meromorphic functions, i.e. its algebraic dimension is zero.

Battaglia and Zaffran (arXiv:1108.1637) computed the basic Betti numbers for the canonical holomorphic foliation on a moment-angle manifold Z corresponding to a shellable fan. They conjectured that the basic cohomology ring in the case of any complete simplicial fan has a description similar to the cohomology ring of a complete simplicial toric variety due to Danilov and Jurkiewicz. We prove the conjecture. The proof uses an Eilenberg-Moore spectral sequence argument; the key ingredient is the formality of the Cartan model for the torus action on Z.

The talk is based on joint works with Hiroaki Ishida, Roman Krutowski, Yuri Ustinovsky and Misha Verbitsky.

Automorphism groups consisting of algebraic elements

A. Perepechko

(IITP RAS; HSE, Russia)

Given an affine algebraic variety X such that the neutral component of its automorphism group consists of algebraic elements, we describe its structure. We also present a result on abstract ind-groups, which is used for this description.

The talk is based on the joint work with Andriy Regeta.

A survey of linear connections with torsion on Lie groups

M. Petrovic
(University of Nis, Serbia)

We consider some special linear connections with torsion on a Lie group. Some curvature properties of these linear connections on a Lie group are given. Finally, we recall some particular examples of Lie groups that might be of particular importance.

The semiconformal curvature tensor on Relativistic spacetimes

N. A. Pundeer
(Jadavpur University, India)

In this talk, our main objective is to investigate the semiconformal curvature tensor on relativistic spacetime. It is shown that the energy-momentum tensor with divergence-free semiconformal curvature tensor is of Codazzi type, on the other hand the energy-momentum tensor of a space-time having semi-symmetric semiconformal curvature tensor is also semi-symmetric. The semiconformal curvature tensor has also been expressed in terms of different tensors already known in the literature, and the association between their divergences has also been established.

N-spin Calogero-Moser model: a superintegrable system

N. Reshetikhin
(Tsinghua University, Beijing, China)

The talk will start with a brief review of superintegrability. Then the N-spin Calogero-Moser model will be introduced and as an example of a superintegrable system. At the end we will discuss how this model is related to 2-dimensional quantum Yang-Mills theory.

Logarithmic R-map

Y. Ruan
(Zhejiang University, China)

The fundamental object of Gromov-Witten theory is the notation of stable map to a target Kahler/Symplectic manifold. In LG A-model in physics, the target carries an additional C^* action called R-charge. It twisted the notation of map in so called gauged linear sigma model. Furthermore, its moduli space is noncompact even with stability condition. In the talk, we will introduce the notation of R-map. The effort to compactify it leads to the theory of Logarithmic R-maps. This is a joint work with Qile Chen and Felix Janda.

The Johannsen-Psaltis Spacetime with Charge

Kh. Saifullah
(Quaid-i-Azam University, Islamabad, Pakistan)

The Kerr spacetime hypothesis can be tested by using two approaches namely the top-bottom approach and bottom-up approach. The first one involves introducing the deviations in the Kerr metric through a theoretical model. The second approach involves introducing the deviations in terms of parameters. The metric proposed by Johannsen and Psaltis is one such parametrically deformed Kerr spacetime. It reduces to the Kerr metric when one sets the deviation parameters to zero. We construct some generalizations of this spacetime including the charged and accelerated versions and discuss their horizon structure and dynamics.

Similarity transformation to explore variable-coefficient nonlinear model equations: dynamics of solitons, breathers, and rogue waves in inhomogeneous media

K. Sakkaravarthi
(Asia Pacific Center for Theoretical Physics (APCTP), South Korea)

The coupled nonlinear Schrödinger type equations with varying dispersion and (self-phase & cross-phase modulations and four-wave mixing) nonlinearities, which govern the dynamics of beam propagation inhomogeneous optical media, are exciting objects of study. We discuss similarity transformation, an efficient yet simplest tool to explore such models for bringing out the dynamical characteristics of their underlying nonlinear waves. In this talk, we briefly investigate the role of modulated nonlinearities in the dynamics of vector optical bright solitons, Akhmediev & Ma breathers, and rogue waves with the help of the explicit analytical solutions. We highlight the possibility of controlling these nonlinear waves revealing different characteristics.

A local index formula for the spectral triple associated with metaplectic operators

A. Savin
(RUDN University, Russia)

We consider the spectral triple (A, H, D). Here A is the algebra of operators on $L^2(\mathbb{R}^n)$ generated by e^{ikx}, shift operators $u(x) \mapsto u(x-a)$ and metaplectic operators associated with isometric linear symplectic operators on $T^*\mathbb{R}^n$; H is the graded Hilbert space of L^2-differential forms on $\mathbb{R}^n$, while D is the unbounded self-adjoint first order operator on H introduced, for instance, by Higson-Kasparov-Trout (1998). Then H has a natural structure of A-module and we show that (A, H, D) is a spectral triple which satisfies the conditions in Connes–Moscovici local index theorem. Our main result is an explicit computation of the Connes–Moscovici residue cocycle. It turns out that each component of the residue cocycle is a cyclic cocycle. Our results are consistent with the results by Farsi, Walters, Watling on noncommutative orbifolds. The talk is based on joint work with Elmar Schröhe (Hannover). The reported study was funded by RFBR and DFG, project number 21-51-12006.

Differential invariants of one parametrical group of transformations

X. Sharipov
(Samarkand State University, Uzbekistan)

In this work differential invariants of Lie group of one parametric transformations of the space of two independent and three dependent variables are studied. It is shown method of construction of invariant differential operator. Obtained results applied for finding differential invariants of surfaces.

Quantising the argument shift method: quasi-derivations and other constructions

G. Sharygin
(Lomonosov Moscow State University, Russia)

Argument shift method is one of the simplest and productive ways to obtain commutative families in a Poisson algebra A. It is closely related to Magri induction and works in a very general situation. On the other hand, raising the corresponding algebras to the quantised version of A is a rather difficult problem; the known solutions in case $A = S(\mathfrak{g})$, where $\mathfrak{g}$ is a finite-dimensional Lie algebra, are based on considerations of infinite-dimensional Lie algebras, such as Yangians or Kac-Moody algebras and give the answer in terms of the generators of the centre of A. In my talk I will suggest a method to define the shift of the argument in the quantisation of the algebra $S(\mathfrak{gl}_n)$, that is on the universal enveloping algebra of the Lie algebra $\mathfrak{gl}_n$. This method is based on the application of "quasi-derivations" of the algebra $U(\mathfrak{gl}_n)$. These operators raise to the universal enveloping algebra partial derivations on $S(\mathfrak{gl}_n)$; they verify a twisted version of the Leibniz rule and we can show that the first derivation in their direction send the centre of $U(\mathfrak{gl}_n)$ to a commutative family. In my talk I will explain the relation of these operators with other constructions on the universal enveloping algebras and Lie groups, I will also relate this construction to a combinatoric question about the representation of the permutation groups.

The talk is based on a joint work with Y. Ikeda.

Quantum representation theory and Manin matrices

A. Silantyev
(Joint Institute for Nuclear Research, State University "Dubna", Russia)

In the end of 80s Yuri Manin proposed to consider quadratic algebras as "quantum linear spaces" (a linear analogue of non-commutative spaces). He introduced a generalisation of the tensor product for the category of quantum linear spaces and proved that this product gives a structure of closed monoidal category. Matrices which describe graded homomorphisms of quadratic algebras (over some algebra) were called "Manin matrices". Quantum Representation Theory is an extension of the "classical" representation theory for the case of quantum representation space. To construct this theory we formulate a general approach to representations in a generalisation of closed monoidal categories, which we call "relatively closed monoidal categories". In the case of the category of graded (super-)algebras equipped with the Manin product we obtain quantum representations on quantum linear (super-)spaces. We describe these representations via Manin matrices.

Integrability criteria for autonomous and non-autonomous second-order differential equations

D. Sinelshchikov

(HSE University, Russia)

In this talk we consider a family of cubic, with respect to the first derivative, second-order differential equations. This family of equations is a projection of two-dimensional geodesics equations. In addition, particular members of the considered family often appear in various applications in mechanics, physics, biology and so on. We study equivalence problems for the considered family of equations and its integrable particular cases, including Painleve-type equations. As equivalence transformations we use generalized nonlocal transformations. It is demonstrated that solutions of these equivalence problems leads to new integrability criteria for the considered class of equations. For each member of the constructed equivalence classes it is possible to obtain the general solution in the parametric form and in the case of autonomous transformations an autonomous first integral in the parametric form. Furthermore, we study the possibility of the existence of a Lax representation for equations from the constructed equivalence classes. In particular, we demonstrate that an equation from this family admits a certain quadratic rational first integral if and only if it possesses a Lax representation with the L matrix from the $sl(2, \mathbb{C})$ algebra. Moreover, we show that the existence of this Lax representation, and, hence, a quadratic first integral, is equivalent to linearizability of the corresponding equation via certain nonlocal transformations. Intrinsic characterization of this subfamily of the considered family can be obtained by calculating compatibility conditions for an overdetermined system of equations on the elements of the L matrix. We explicitly find these conditions in some particular cases and also illustrate our results by several examples.

Feynman checkers: Minkowskian lattice quantum field theory

M. Skopenkov

(HSE University and IITP RAS, Russia)

We present a new completely elementary model which describes creation, annihilation and motion of non-interacting electrons and positrons along a line. It is a modification of the model known under the names Feynman checkers, or one-dimensional quantum walk, or Ising model at imaginary temperature. The discrete model is consistent with the continuum quantum field theory, namely, reproduces the known expected charge density as lattice step tends to zero. It is exactly solvable in terms of hypergeometric functions.

Short SL_2-structures on simple Lie algebras

R. Stasenko

(Lomonosov Moscow State University, Russia)

There is the classical Tits-Kantor-Koeher construction, which allows one to construct from a simple Jordan algebra J a simple Lie algebra $\mathfrak{g}$, having the form:

$$\mathfrak{g} = \mathfrak{Der}(J) \oplus \mathfrak{sl}_2(J).$$

The Tits-Kantor-Koeher theorem states that there is a one-to-one correspondence between simple Jordan algebras and simple Lie algebras equipped with decomposition as above.

The Tits-Kantor-Koeher construction can be interpreted as a linear representation of the Lie algebra $\mathfrak{sl}_2$ by automorphisms of the Lie algebra $\mathfrak{g}$ which decomposes into irreducible representations of dimensions 1 and 3. The following concept is a natural generalization of this construction. Let S be a reductive algebraic group and let $\mathfrak{g}$ be a Lie algebra. The homomorphism $\Phi : S \to \mathrm{Aut}(\mathfrak{g})$ is called the S-structure on the Lie algebra $\mathfrak{g}$.

The talk is devoted to a SL_2-structures on simple Lie algebras. We call SL_2-structure *short* if the representation Φ of the group SL_2 decomposes into irreducible representations of dimensions 1, 2 and 3. In this case, the isotypic decomposition of the representation Φ have the form:

$$\mathfrak{g} = \mathfrak{g}_0 \oplus \big(\mathbb{C}^2 \otimes J_1\big) \oplus \big(\mathfrak{sl}_2 \otimes J_2\big).$$

The Tits-Kantor-Koeher construction corresponds to the case $J_1 = 0$. The talk deals with the case of $J_1 \neq 0$.

Similarly to the Tits-Kantor-Koeher theorem, there is a one-to-one correspondence between simple Lie algebras with a short SL_2-structure with $J_1 \neq 0$ and so-called simple symplectic Lie–Jordan structures $(J_1; J_2; \mathfrak{g}_0; \delta_0)$, where J_1 is a symplectic space, $\mathfrak{g}_0$ is a reductive Lie subalgebra in $\mathfrak{sp}(J_1)$, J_2 is a simple Jordan subalgebra of symmetric operators on J_1 and δ_0 is some symmetric bilinear map. A complete classification of short SL_2-structures on simple Lie algebras will be given.

Super Yangians and quantum loop superalgebras

V. Stukopin

(Moscow Institute of Physics and Technology, Russia)

We consider relations between super Yangians and quantum loop superalgebras. We classify Hopf superalgebra structure and describe relation between categories of representations of super Yangian and quantum loop superalgebra.

On matrix sets with special properties

O. Styrt

(MSU, MIPT, Russia)

Subsets of a matrix algebra over a field that are invariant under conjugation and contain the linear span of each two of their commuting elements are described. They obviously include the subsets of semisimple and nilpotent matrices. In the paper, the case of an algebraically closed field is considered. The problem is easily reduced to description of subsets of semisimple matrices and subsets of nilpotent matrices with the given properties. So, in the semisimple case there are four of such subsets. As for the nilpotent case, it is proved that the subset should be defined by the condition that the sizes of all Jordan cells of the matrix belong to a certain number set. An explicit criterion is obtained in terms of this set.

Quantum differentiation and integration for the quantum plane

F. Sukochev

(University of New South Wales, Kensington, Australia)

We explain recent results concerning (quantum) differentials and integrals on the noncommutative (Moyal) plane. We give full characterisation of elements on the noncommutative (Moyal) plane such that their quantum derivatives belong to the weak Schatten class $\mathcal{L}_{d,\infty}$, which means that these derivatives are d-times integrable in the quantum integration sense. We then calculate the quantum integration of these derivatives by adapting Connes' integration formulae to the noncommutative (Moyal) plane. This is a joint work with E. McDonald and X. Xiong.

Closed geodesics on non-simply-connected manifolds

I. Taimanov

(Sobolev Institute of Mathematics, Novosibirsk, Russia)

We discuss the problem on the existence of closed geodesics on non-simply-connected manifolds and expose some recent results on the existence of infinitely many closed geodesics on certain connected sums and general three-manifolds and for generic Finsler metrics. These results are obtained jointly with H.-B. Rademacher.

Electrical networks and positive lagrangian grassmanian

D. Talalaev

(Lomonosov Moscow State University, Russia)

The dimer model, the Ising model, and the electric network model are in some sense archetypal examples of integrable models of statistical physics on weighted graphs. They correspond to general linear, orthogonal, and symplectic structure groups. In particular, the moduli spaces of such systems are embedded in completely non-negative Grassmannians: complete, orthogonal and Lagrangian, respectively. The talk will focus on the latter case, which corresponds to the problem of electrical networks. This statement in different forms and with the help of different techniques was constructed in the works of Lam, Chepuri, George, Speyer, as well as Bychkov, Gorbunov, Kazakov and the speaker arXiv:2109.13952.

On homotopy braids

V. Vershinin

(Universite de Montpellier, Montpellier, France)

The homotopy braid group $\widehat{B}_n$ is the subject of the talk. First, linearity of $\widehat{B}_n$ over the integers is proved. Then we prove that the group $\widehat{B}_3$ is torsion free.

Critical metrics on 4-manifolds with harmonic anti-self dual Weyl tensor

E. Viana

(Instituto Federal de Educação, Ciência e Tecnologia do Ceará - IFCE, Brasil)

In this talk, we study 4-dimensional simply connected, compact critical metric of the volume functional with harmonic anti-self dual Weyl tensor. We show that a 4-dimensional simply connected, compact critical metric of the volume functional with harmonic anti-self dual Weyl tensor and satisfying a suitable pinching condition is isometric to a geodesic ball in a simply connected space form $\mathbb{R}^4$, $\mathbb{H}^4$ or $\mathbb{S}^4$. https://doi.org/10.1016/j.geomphys.2021.104434

Donagi-Witten construction and a graded covering of a supermanifold

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In geometry there is a well-known notion of a covering space. A classical example is the following universal covering $p : \mathbb{R} \to S^1$, $t \mapsto \exp(it)$. In algebra there are also notions of a covering or a cover. An example of a cover in algebra is a flat cover in the theory of modules. Existence of a flat cover proved simultaneously by Bican, El Bashir and Enochs (2001). Another example is a torsion-free cover of a module over an integral domain. All these cover(ing)s satisfy some common universal properties.

In the paper "Super Atiyah classes and obstructions to splitting of supermoduli space", Donagi and Witten suggested a construction of a first obstruction class for splitting a supermanifold via differential operators. We realized that an infinite prolongation of this construction is a graded covering of a supermanifold. In other words it is a covering of a supermanifold in the category of graded manifolds. We will discuss a particular (simple) case of our construction: the case of a Lie supergroup and a Lie superalgebra.

C^*-algebraic approach to the principal symbol

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The principal symbol mapping as a homomorphism of C^*-algebras was introduced by Sukochev-Zanin (in $\mathbb{R}^d$) and McDonald-Sukochev-Zanin (in a number of commutative and noncommutative examples). Those examples, however, do not cover the case of an arbitrary Riemannian manifold. We show the equivariant behaviour of the principal symbol mapping with respect to diffeomorphisms of $\mathbb{R}^d$. This allows to construct of a principal symbol mapping for compact Riemannian manifolds. As an application, we prove the Connes Trace Formula. This is a joint work with F. Sukochev.

Deformations of Dirac operators and applications

W. Zhang
(Nankai University, Tianjin, China)

Index theory for Dirac type operators as well as their deformations have played important roles in many problems in geometry and topology. We will survey some of these applications in the talk.
